Exercise: An Example of Euler's Method

Exercise. Consider the first order ODE $y' = f(x, y) = y^2 - x$ with initial condition $y(0) = -1$. Estimate $y(1)$, using Euler's method with $h = 0.5$. Organize your answer in a table.

$$\frac{dy}{dx} = y^2 - x, \quad y(0) = -1$$

$$y(0.5) = y(0) + h\, y'(0, -1)$$

$$= -1 + 0.5(1)$$

$$= -0.5$$

$$y(1) = y(0.5) + h\, y'(0.5, -0.5)$$

$$= -0.5 + 0.5(0.25 - 0.5)$$

$$= -0.5 - 0.125$$

$$= -0.625$$

18.03SC Practice Problems 3

Euler's method

[Euler's method]

1. Use Euler's method to estimate the value at $x = 1.5$ of the solution of $\frac{dy}{dx} = F(x,y) = y^2 - x^2$ for which $y(0) = -1$. Use step size $h = 0.5$. Recall the notation $x_0 = 0$, $y_0 = -1$, $x_{n+1} = x_n + h$, $y_{n+1} = y_n + m_n h$, $m_n = F(x_n, y_n)$. Make a table with columns n, x_n, y_n, m_n, $m_n h$. Draw the Euler polygon.

2. Is the estimate found in Question 1 likely to be too large or too small?

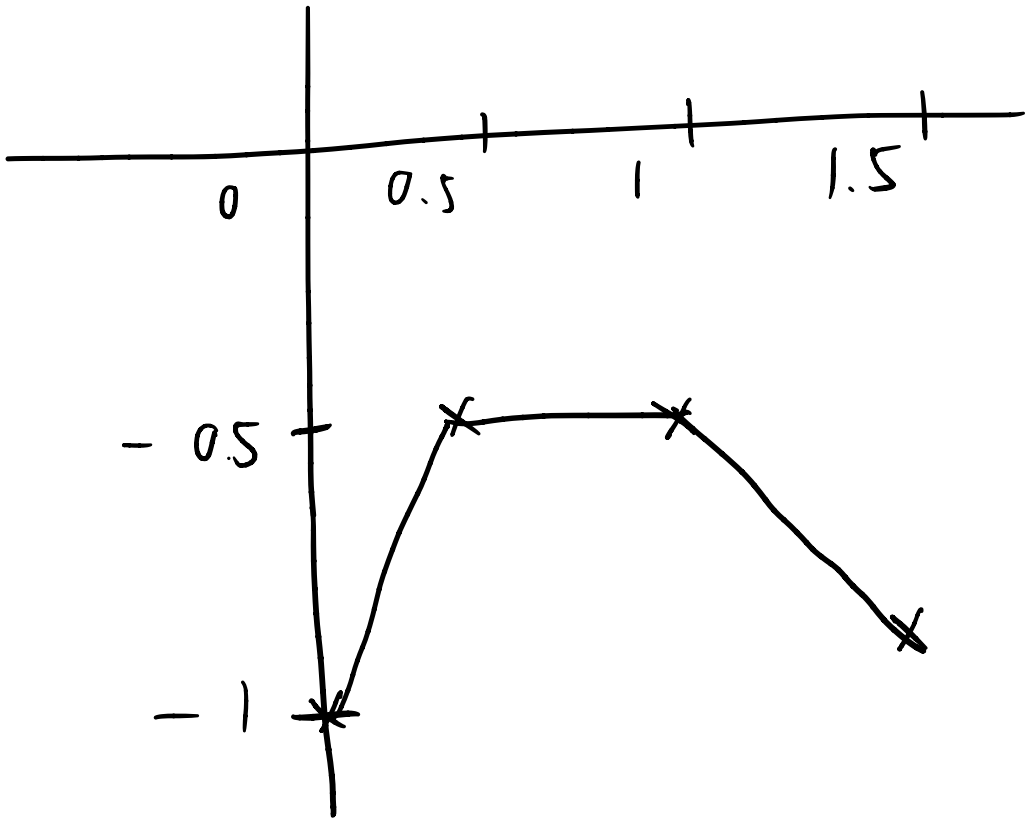
1.

4/5/25

$$\frac{dy}{dx} = y^2 - x^2 \quad , \quad y(0) = -1$$

n	x_n	y_n	m_n	$m_n h$
0	0	-1	1	0.5
1	0.5	-0.5	0	0
2	1	-0.5	-0.75	-0.375
3	1.5	-0.875	-1.484	-0.742

$\therefore y = -0.875$



2. $\frac{dy}{dx} = y^2 - x^2$

$$\frac{d^2y}{dx^2} = 2y\frac{dy}{dx} - 2x$$

$n = 0 \Rightarrow \frac{d^2y}{dx^2} = 2(-1)(1) - 2(0) = -2$

$n = 2 \Rightarrow \frac{d^2y}{dx^2} = 2(-0.5)(-0.75) - 2(1) = -1.25$

$\therefore$ likely too large